Hydrogen Stark Broadening by Ion Impacts on Moving Emitters

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The "classical path theory" for the plasma broadening of hydrogen lines calls for a closer examination of the influence of radiator motion on Stark profiles. For a radiator with constant velocity Stark broadening is effected by an anisotropic plasma, and in a frame of reference moving with the emitter the line shape depends not only on the radiator speed but on the direction of emission as well. From this, a somewhat intricate correlation of Stark and Doppler broadening arises which has to be taken into account in the calculation of the full line profile. As an example, broadening by plasma ions is investigated in the impact limit where a distinct dependence of Stark broadening on the emitter velocity is found especially for very heavy, immovable ions.

1. Introduction

Ever since 1919, when Holtsmark's much-quoted paper [1] was published, hydrogen Stark profiles have been calculated with the help of the "static ions approximation", i.e., neglecting fluctuations of the low frequency ionic part of the plasma microfield. The admissibility of this approximation seemed to be well secured for typical laboratory plasmas [2], and for that reason theoretical research in the field has concentrated mainly on electron broadening. The greater was the surprise when a few years ago Kelleher and Wiese [3] and Wiese et al. [4] proved experimentally that the central parts of the first Balmer lines depend markedly on the reduced mass of the radiating atom-perturbing ion pair. From this the conclusion was drawn that relative atom-ion motion does play its part in hydrogen Stark broadening despite of the theoretical estimates. Other more recent experiments give strong support to this conclusion: Still more pronounced discrepancies between theoretical profiles (obtained with the static ions approximation) and measured ones have been found in the Lyman line centres by Grützmacher and Wende [5–8], and for the Balmer lines at low plasma densities by Ehrich and Kelleher [9, 10] and Ehrich [11].

Challenged by these experimental results, various authors have elaborated improvements of the theory by approximately taking account of "ion dynamical effects". Most of the approaches are confined to L_α from the outset [12–15] or do not work satisfactorily for other lines [16], but the results of "model microfield calculations" for both Lyman and Balmer lines [17, 18] confirm that indeed the agreement of theory and experiment is greatly improved if the former allows for relative emitter-ion motion. Thus it might appear that only more detailed questions are left to be answered, concerning for example the role of low frequency electronic microfields which has been emphasized by Griem [14, 15] in the case of L_α. This appearance is deceptive, however, as none of the papers just mentioned gives a proper general treatment of the influence of radiator motion on hydrogen Stark broadening and the correlation with Doppler broadening which results from it. Frequently, statements of this problem are either vague or completely missing [12–14, 16, 19], and among the theoretical approaches cited above it is only Griem's recent treatment of the L_α profile from dense plasmas [15] which tries to account approximately for this effect.

The usual artifice to circumvent this additional difficulty is to consider — without further justification — an atom at rest in a gas of fictious ions to which the reduced mass μ of the atom-ion pair in question is ascribed [12, 13, 16, 19]. By this, Stark broadening is assumed to be the same for all radiators irrespective of their velocities, and Stark profiles calculated in this way depend on the reduced mass μ *a priori*, just as it has been found experimentally. Theoretically, however, it is hard to see whether this procedure provides a sound approximation or not. Doubts arise especially for the broadening of hydrogen lines by heavy ions, Ar^+ for instance, where the intuitive picture is the

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reverse one, namely a radiator flying through an assembly of immovable ions. A similar criticism applies to the model microfield treatment [17, 18] where one and the same microfield autocorrelation function has been used for all atoms to simplify the calculations. Therefore, it is still not clear whether an improved theory would really bring the reduced mass μ into play. Furthermore, any statistical correlation of Stark and Doppler broadening is neglected in this way; the full line profile is calculated by simply convolving the Stark profile with a Gaussian Doppler profile.

It is the purpose of the present paper to call further attention to this point by means of a simple example. Based on the fact that hydrogen line profiles must in particular depend on radiator motion whenever they are affected by relative radiator-ion motion, Sect. 2 gives a brief survey of the appropriate "classical path" theory of line broadening [20, 21]. For an atom moving with constant velocity Stark broadening is caused by an anisotropic plasma. For that reason, it is necessary to define two "speed dependent Stark profiles" (corresponding to the emission of light perpendicular or parallel to the radiator velocity) from which the full line profile (including Doppler broadening) may be calculated. To exemplify this Sects. 3 to 5 contain an investigation of hydrogen Stark broadening by ion impacts on moving emitters. As impact broadening is purely dynamical, effects of emitter motion become most obvious in this case without too complicated calculations, and a distinct dependence of Stark broadening on the radiator velocity is found. Practically this result is rather insignificant, as a plasma must have extremely low density (or very high temperature) in order that the impact theory is valid for ions in the line core — and then Doppler broadening is of overwhelming importance anyhow. Nevertheless, the result should make the theory reconsider the problem of "ion dynamical effects" on hydrogen lines without shirking from the difficulties brought about by radiator motion. This is one of the conclusions drawn in Section 6. Some calculational items from Sect. 4 have been deferred to an appendix.

2. Hydrogen Stark Broadening for Moving Emitters

For hydrogen Stark broadening in a plasma of very low density, the "unified theory" is valid not only for the electrons, but also for the ions [19]. In that case, the line centre may be calculated rather easily utilizing the "impact theory" [2] for electrons and ions. Still there is an important difference between electron and ion broadening, provided that the electron temperature is at least of the same order of magnitude as the heavy particle temperature. Under this condition, the relative velocity in an atom-electron collision is practically always the electron velocity, and with regard to Stark broadening by electrons the atomic velocity may be neglected. This is not true, however, for atom-ion collisions — with ions of large mass like Ar^+, for example, the relative velocity is, on the contrary, close to the atomic velocity. Therefore, the impact theory for ions has to take atomic motion into account and cannot be elaborated in complete analogy to the impact theory for electrons. A similar conclusion holds at higher plasma densities where ion broadening is no longer impact broadening, if the line profiles are affected by relative emitter-ion motion.

Owing to the "accidental" degeneracy of the energy levels involved, the general theory of hydrogen line broadening including radiator motion is very complicated even in the impact limit [20, 21] and under the dipole interaction approximation [22, 23]. Fortunately, it turns out that the "classical path approximation" is sufficient for this case [24, 25]. (The usual discussion, with the atom at rest, relates to perturber trajectories only, but classical perturber trajectories imply classical emitter motion as well.) For that reason, it is assumed in what follows that the radiating atom has constant translational velocity, as have the perturbers which are taken to be classical charged point particles; only the internal states of the radiator are treated by quantum mechanics. Essentially the same model has been used by Ward et al. [26] in the frame of a classical theory for spectral line broadening.

With the approximations stated, the fundamental formula for the intensity of radiation emitted with (angular) frequency ω into a direction specified by the unit vector $\boldsymbol{k}$ is [20] (except for the normalization constant)

$$I(\omega;\boldsymbol{k}) \propto \sum_{\lambda} \int_{-\infty}^{\infty} \mathrm{d}t\, e^{i\omega t} \int \mathrm{d}^3 V f(\mathbf{V}) \cdot \mathrm{Tr_a}\{d^{(\lambda)} \langle d^{(\lambda)}(t|\mathbf{V})\rangle_{\mathrm{p}}\, \varrho_{\mathrm{a}}\}\, e^{-i\boldsymbol{k}\cdot\mathbf{V}t}. \quad (2.1)$$

Here, $\mathbf{V}$ denotes the atomic velocity which is distributed according to $f(\mathbf{V})$, $\mathrm{Tr_a}$ is the trace over internal atomic states (with density operator ϱ_a), and $d^{(\lambda)}$, $\lambda = 1, 2$, are components of the atomic dipole moment perpendicular to the radiation wave vector, $\mathbf{k} = \omega \boldsymbol{k}/c$. The Heisenberg operator $\mathbf{d}(t|\mathbf{V})$ describes the temporal evolution of the dipole moment for an atom with velocity $\mathbf{V}$ under the influence of the charged plasma particles, and $\langle . \rangle_p$ denotes the average over all atom-plasma interactions to be found.

Formula (2.1) differs in some respects from the familiar form used for an atom at rest [2]. Most conspicuous are the extra factor $e^{-i\boldsymbol{k}\cdot\mathbf{V}t}$ and the average over atomic velocities which lead to the pure Doppler profile for vanishing Stark broadening, that is $\mathbf{d}(t|\mathbf{V}) = \mathbf{d}\, e^{-i\omega_0 t}$ (ω_0 frequency of the unperturbed line). Moreover, (2.1) still allows explicitly for a dependence of I on the direction of emission $\boldsymbol{k}$ which might be brought about by an anisotropy of the plasma due to external fields, for example. The present treatment, however, will be confined to homogeneous, spherically symmetric plasmas with a Maxwell distribution for the atomic velocities:

$$f(\mathbf{V}) = f_M(V) = (\pi V_0^2)^{-3/2} \exp(-V^2/V_0^2), \quad V_0^2 = 2kT/M \tag{2.2}$$

(M is the atomic mass, and we assume that there is one and the same temperature T for all particle species). Beside this, we neglect any dependence of ϱ_a on the direction of $\mathbf{V}$ and resign to the case of isotropic excitation. (For the impact treatment in the following sections we shall assume that ϱ_a is completely independent of $\mathbf{V}$, as all previous investigations have done [2], but at present the weaker assumption is sufficient.)

Then, the intensity $I(\omega; \boldsymbol{k})$ is the same for all directions of emission, and it is tempting to conclude [20] that *any* two orthogonal components of the atomic dipole moment may be used in (2.1), which would amount to the replacement of

$$\sum_\lambda d^{(\lambda)} \langle d^{(\lambda)}(t|\mathbf{V})\rangle_p \quad \text{by} \quad \tfrac{2}{3}\, \mathbf{d} \cdot \langle \mathbf{d}(t|\mathbf{V})\rangle_p .$$

Yet, this conclusion is fallacious. To obtain the correct expression we write (2.1) in the form

$$I(\omega) \propto \int_{-\infty}^{\infty} \mathrm{d}t\, e^{i\omega t} \int \mathrm{d}^3 V f_M(V) \cdot e^{-i\boldsymbol{k}\cdot\mathbf{V}t}\, \mathrm{Tr_a}\{[\mathbf{d} \cdot \langle \mathbf{d}(t|\mathbf{V})\rangle_p - \boldsymbol{k}\cdot\mathbf{d}\, \langle \boldsymbol{k}\cdot\mathbf{d}(t|\mathbf{V})\rangle_p]\, \varrho_a\}. \tag{2.3}$$

Here, $\mathrm{Tr_a}\{\mathbf{d}\cdot\langle\mathbf{d}(t|\mathbf{V})\rangle_p\, \varrho_a\}$ actually does not depend on the direction of $\mathbf{V}$ under the assumptions we have made, but only on its modulus, the speed V. Therefore, the first part of the trace in (2.3) may be evaluated at once taking $\mathbf{V} = V\boldsymbol{e}_z$. However, if we rotate $\mathbf{V}$ into the z-axis in the second part, too, we have to be careful to apply the same rotation to $\boldsymbol{k}$ as well. Subsequently performing the integration over the azimuth of $\mathbf{V}$ and making use of

$$\mathrm{Tr_a}\{d_x \langle d_x(t|V\boldsymbol{e}_z)\rangle_p\, \varrho_a\} = \mathrm{Tr_a}\{d_y \langle d_y(t|V\boldsymbol{e}_z)\rangle_p\, \varrho_a\},$$

we obtain

$$I(\omega) \propto \int_{-\infty}^{\infty} \mathrm{d}t\, e^{i\omega t} \int_0^{\infty} \mathrm{d}V\, V^2 f_M(V) \cdot \int_{-1}^{1} \mathrm{d}\eta\, e^{-ikV\eta t}\, \mathrm{Tr_a}\{[(1+\eta^2)\, d_x \langle d_x(t|V\boldsymbol{e}_z)\rangle_p + (1-\eta^2)\, d_z \langle d_z(t|V\boldsymbol{e}_z)\rangle_p]\, \varrho_a\} \tag{2.4}$$

with $\eta = \cos\vartheta$ and ϑ the angle formed by $\boldsymbol{k}$ and $\mathbf{V}$.

To get a more transparent form of $I(\omega)$ we separate the effects of Stark and Doppler broadening as far as possible by defining two "speed dependent Stark profiles" which correspond to the emission of radiation parallel ($\|$) and perpendicular ($\perp$) to the atomic velocity ($\mathbf{V} = V\boldsymbol{e}_z$ here), respectively, in a frame of reference moving with the atom:

$$I_S^{\|}(\omega|V) = \frac{3}{2\pi\, \mathrm{Tr_a}(\mathbf{d}\cdot\mathbf{d}\, \varrho_a)} \cdot \int_{-\infty}^{\infty} \mathrm{d}t\, e^{i\omega t}\, \mathrm{Tr_a}\{d_x \{d_x(t|V\boldsymbol{e}_z)\rangle_p\, \varrho_a\},$$

$$I_S^{\perp}(\omega|V) = \frac{3}{4\pi\, \mathrm{Tr_a}(\mathbf{d}\cdot\mathbf{d}\, \varrho_a)} \cdot \int_{-\infty}^{\infty} \mathrm{d}t\, e^{i\omega t}\, \mathrm{Tr_a}\{[d_x \langle d_x(t|V\boldsymbol{e}_z)\rangle_p + d_z \langle d_z(t|V\boldsymbol{e}_z)\rangle_p]\, \varrho_a\}. \tag{2.5}$$

In general, $I_S^{\|}$ and $I_S^{\perp}$ are different because an atom with velocity $\mathbf{V} \neq \mathbf{0}$ „sees" a plasma which is anisotropic. Hence, the time evolution of the components of $\langle\mathbf{d}(t|\mathbf{V})\rangle_p$, parallel or perpendicular to $\mathbf{V}$, is not the same. By comparison with (2.4), the full line profile is found to be

$$I(\omega) = 2\pi \int_0^{\infty} \mathrm{d}V\, V^2 f_M(V) \cdot \int_{-1}^{1} \mathrm{d}\eta\, [\eta^2 I_S^{\|}(\omega - kV\eta|V) + (1-\eta^2)\, I_S^{\perp}(\omega - kV\eta|V)] \tag{2.6}$$

where k may be taken to be $k_0 = \omega_0/c$ for a spectral *line* with central frequency ω_0 (neglect of Stark broadening in the calculation of the Doppler shift).

Under the assumptions we have made, (2.6) gives the correct expression for the profile of a spectral line emitted by moving radiators. It shows that the final line profile including Stark and Doppler broadening has to be calculated in a rather complicated way from the Stark profiles $I_S^{\parallel}$ and $I_S^{\perp}$ which depend on the radiator speed V and have to be taken at the Doppler shifted frequency corresponding to the velocity component $V\eta$ in the direction of emission. The weight factors η^2 and $1-\eta^2$ in (2.6) take care that the maximal Doppler shift $\pm kV$ is observed only for radiation parallel or antiparallel to $\mathbf{V}$, while emission without Doppler shift $(\eta = 0)$ has to occur perpendicular to the atomic velocity.

All this leads to an intricate coupling of Stark and Doppler broadening and, in general, (2.6) cannot be simplified further. It is only in special cases, that this expression reduces to a more familiar form. First of all, $I(\omega)$ is obtained from the well known convolution of Doppler and Stark profiles whenever the velocity dependence of Stark broadening is completely negligible. This is true for perturbers moving much faster than the typical radiator (broadening by electrons) as well as under the "static ions approximation" (which has often been used for the ion broadening of hydrogen lines [2]). Then we have $I_S^{\parallel}(\omega | V) = I_S^{\perp}(\omega | V) = I_S(\omega)$, and (2.6) becomes

$$I(\omega) = 2\pi \int_0^\infty \mathrm{d}V\, V^2 f_M(V) \int_{-1}^{1} \mathrm{d}\eta\, I_S(\omega - k_0 V \eta) = \int_{-\infty}^{\infty} \mathrm{d}\omega'\, I_S(\omega - \omega') \frac{c}{\pi^{1/2}\, \omega_0\, V_0} \cdot \exp\{-(c\,\omega'/V_0\, \omega_0)^2\}\,. \tag{2.7}$$

A similar simplification results from the assumption that Stark broadening depends on the radiator speed V only, independent of the direction of emission [27]. This amounts to setting $I_S^{\parallel}(\omega | V) = I_S^{\perp}(\omega | V) = I_S(\omega | V)$ and makes it possible to write

$$I(\omega) = 4\pi \int_0^\infty \mathrm{d}V\, V^2 f_M(V) \cdot \int_{-\infty}^{\infty} \mathrm{d}\omega'\, I_D(\omega - \omega' | V)\, I_S(\omega' | V) \tag{2.8}$$

with the normalized Doppler profile for radiator speed V defined to be the rectangular profile

$$I_D(\omega | V) = \frac{c}{2\omega_0 V}\, \Theta\left(\frac{\omega_0 V}{c} - |\omega|\right). \tag{2.9}$$

At last, there are two cases of special practical interest: preponderance of either Stark or Doppler broadening. Stark broadening predominates if we have $I_S^{\parallel,\perp}(\omega - kV\eta | V) \simeq I_S^{\parallel,\perp}(\omega | V)$ for $|\eta| \leqq 1$ and all values of V which contribute significantly to (2.6) according to the Maxwell distribution $4\pi V^2 f_M(V)$ of atomic speeds. By this, the η-integration in (2.6) becomes trivial:

$$I(\omega) = 4\pi \int_0^\infty \mathrm{d}V\, V^2 f_M(V)\, I_S(\omega | V) \tag{2.10}$$

with

$$I_S(\omega | V) = \tfrac{1}{3} I_S^{\parallel}(\omega | V) + \tfrac{2}{3} I_S^{\perp}(\omega | V) = \frac{1}{2\pi\, \mathrm{Tr_a}(\mathbf{d} \cdot \mathbf{d}\, \varrho_a)} \int_{-\infty}^{\infty} \mathrm{d}t\, e^{i\omega t}\, \mathrm{Tr_a}\{\mathbf{d} \cdot \langle \mathbf{d}(t|\mathbf{V})\rangle_p\, \varrho_a\}\,. \tag{2.11}$$

It should be noted that once more knowledge of $\mathbf{d} \cdot \langle \mathbf{d}(t|\mathbf{V})\rangle_p$ is sufficient in this case, though we may have $I_S^{\parallel} \neq I_S^{\perp}$ here. Finally, at low plasma densities and/or high temperatures, Doppler broadening is much more important than Stark broadening at least in the line core. Accordingly, both Stark widths may be neglected:

$$I_S^{\parallel}(\omega | V) \simeq I_S^{\perp}(\omega | V) \simeq \delta(\omega - \omega_0)\,.$$

By that, $I(\omega)$ becomes the usual pure Gaussian Doppler profile.

Strictly, however, we are forced to calculate the speed dependent Stark profiles $I_S^{\parallel}(\omega | V)$ and $I_S^{\perp}(\omega | V)$ to be used in (2.6). Commonly, numerical computations will be necessary to evaluate these profiles, and the correct consideration of emitter motion will be much more expensive than the use of some mean Stark profile together with (2.7). Because of this, it seems to be appropriate to study the simple impact limit more closely, for which approximate expressions may be obtained analytically which exemplify the general conclusions drawn up to now.

3. Basic Impact Theory for Moving Emitters

The speed dependent Stark profiles $I_S^{\parallel}$ and $I_S^{\perp}$ defined in (2.5) are the fundamental quantities we are interested in. Except for the velocity dependence of $\langle \mathbf{d}(t | \mathbf{V})\rangle_p$ they have the same general form as the usual Stark profile for an emitter at rest, so their evaluation is not essentially different. Here we want to accomplish this task with the help of the "impact theory" [20, 28, 29] for both electrons and ions. It has already been pointed out that radiator motion does play a part in atom-ion collisions, while it is of minor importance for atom-electron collisions. There is another difference between electron and ion impacts, again due to the fact that electrons move much faster than ions in a thermal plasma. Just like the radiating atom, the ions practically do not move during the time the average electron needs to complete a collision. Therefore, electron impacts take place under the influence of a static ion microfield (which is, however, usually neglected in calculations [2, 30]). On the other hand, there are many electron impacts during a single atom-ion collision which give rise to an "effective" non-hermitian contribution to the Hamilton operator [29]. Fortunately enough, we do not have to discuss these complications in more detail: At very low plasma densities where ion broadening indeed is impact (or "dynamical") broadening, it is much more important in the line centre than electron broadening. This is already indicated by the model microfield results at low densities [17, 18], and it will show up once more at the end of the next section. For that reason, we shall completely neglect electron broadening.

One further item has to be mentioned before we can go on. In the impact approximation, Stark widths are proportional to the perturber density, and at very low densities they will eventually become smaller than even the fine structure splitting of the hydrogen lines as Ehrich and Kelleher [9] have emphasized. In that case, it is certainly incorrect to adhere to a nonrelativistic description of the hydrogen atom with completely degenerate levels. On the other hand, the present paper merely intends to give a simple example of speed dependent Stark broadening. This is why we neglect fine structure splitting, too. Accordingly, the calculations to follow will only have model character with respect to this point.

With these approximations, the evaluation of $I_S^{\parallel}$ and $I_S^{\perp}$ starts in the same way as the calculation neglecting radiator motion [2, 28, 29]. For the present purpose it is sufficient to study the Lyman lines. Then, making use of the "no quenching approximation", there is no perturbation of the lower state, and the important quantity is the "impact operator" for the states of the upper level,

$$\begin{aligned}\Phi(\mathbf{V}) &= \sum_j w_j (S_j - 1) \\ &= \sum_j w_j [U(\infty, -\infty; j) - 1], \end{aligned} \tag{3.1}$$

which determines the mean evolution operator by $\langle U(t|\mathbf{V})\rangle_p = \exp\{\Phi(\mathbf{V})t\}$. S_j is the S-matrix (the evolution operator from time $-\infty$ to ∞) describing the effects of an atom-perturber collision of type "j" on the states of the upper level, and w_j is the probability to have one such collision per unit time.

The various types of collisions may be characterized in different ways. Assuming that the atom starts with velocity $\mathbf{V}$ from position $\mathbf{R}(0) = \mathbf{0}$ at time 0, we may choose the perturber velocity $\mathbf{u}$ and position $\mathbf{r}_0 = \mathbf{r}(0)$ at that time. For the perturber position relative to the atom, this gives

$$\mathbf{r}(t) = \mathbf{r}_0 + (\mathbf{u} - \mathbf{V})\, t \tag{3.2}$$

at time t. A more expedient characterization is by the relative velocity $\mathbf{v} = \mathbf{u} - \mathbf{V}$, the impact parameter $\boldsymbol{\rho}$ (a vector perpendicular to $\mathbf{v}$), and the time of closest approach t_0 (such that $\mathbf{r}(t_0) = \boldsymbol{\rho}$). Then we have

$$\mathbf{r}(t) = \boldsymbol{\rho} + \mathbf{v}(t - t_0)\,, \tag{3.3}$$

and the connexion between (3.2) and (3.3) is given by

$$\begin{aligned}\boldsymbol{\rho} &= \mathbf{r}_0 - (\mathbf{v}\cdot\mathbf{r}_0)\, v^{-2}\, \mathbf{v}\,, \\ t_0 &= -\,\mathbf{v}\cdot\mathbf{r}_0\, v^{-2}\,. \end{aligned} \tag{3.4}$$

For a homogeneous plasma where the ions have density N and a velocity distribution $f_i(\mathbf{u})$, the probability density for collisions labelled "$\boldsymbol{\rho}, \mathbf{v}, t_0$" is

$$\begin{aligned} w(\boldsymbol{\rho}, \mathbf{v}, t_0) = N \int \mathrm{d}^3 r_0 \int \mathrm{d}^3 u\, f_i(\mathbf{u})\, \delta(\mathbf{r}_0 + \mathbf{v}\, t_0 - \boldsymbol{\rho}) \\ \cdot\, \delta(\mathbf{u} - \mathbf{V} - \mathbf{v})\, \delta(t_0 + \mathbf{v}\cdot\mathbf{r}_0\, v^{-2})\,. \end{aligned} \tag{3.5}$$

The integrations are easily done, resulting in

$$w(\boldsymbol{\rho}, \mathbf{v}, t_0) = N\, v\, \varrho^{-1} f_i(\mathbf{v} + \mathbf{V})\, \delta(\hat{\varrho}\cdot \boldsymbol{v}) \tag{3.6}$$

with the δ-function taking care that the impact parameter is at a right angle to the relative velocity ($\hat{\varrho}$ and $\boldsymbol{v}$ are unit vectors along $\boldsymbol{\rho}$ and $\mathbf{v}$). Even for an isotropic plasma with $f_i(\mathbf{u}) = f_i(u)$, w is anisotropic which makes the evaluation of (3.1) slightly more complicated than for a radiator at rest. Still, however, we have the useful symmetry relation $w(-\boldsymbol{\rho}, \mathbf{v}, t_0) = w(\boldsymbol{\rho}, \mathbf{v}, t_0)$.

Inserting (3.6) into (3.1), we obtain the general form of the impact operator for a moving radiator in the classical path approximation:

$$\Phi(\mathbf{V}) = N \int d^3\varrho\, \varrho^{-1} \int d^3v\, v\, f_i(\mathbf{v}+\mathbf{V}) \cdot \delta(\hat{\varrho}\cdot\boldsymbol{v})[U(\infty,-\infty;\boldsymbol{\rho},\mathbf{v})-1]. \quad (3.7)$$

Generalized to include lower state interaction as well, this is just Berman's "straight line path limit" result [31], except for the consequent use of $\boldsymbol{\rho}$ and $\mathbf{v}$ as variables in (3.7) which facilitates the further evaluation of this expression.

If the impact theory is to give the line profile out to frequencies $\Delta\omega$ beyond the half width, the S-matrix treatment entails as validity condition [2], that all collisions which are important in (3.7) have to be completed in the time of interest $1/\Delta\omega$, i.e., they must comply with $v/\varrho \gg \Delta\omega$.

4. Approximate Calculation of the Impact Operator

Though it is possible, under the dipole interaction approximation, to find the exact expression for the evolution operator [32], its use in (3.7) would lead to complicated integrals and obscure rather than elucidate the dependence of Φ on $\mathbf{V}$. That is why we resign to perturbation theory for the evaluation of $U(\infty,-\infty;\boldsymbol{\rho},\mathbf{v})$ [28, 29] which will yield more transparent results. For the calculation, it is convenient to switch to an interaction picture where the unperturbed oscillations of the atomic states have been split off. In what follows, we shall assume that this has been done, but we shall not introduce new symbols for the various quantities. Instead, we replace ω by $\Delta\omega=\omega-\omega_0$, the frequency distance from the unperturbed line, in the right hand sides of (2.5).

With dipole interaction only, and to second order of perturbation theory, the evolution operator is

$$U(\infty,-\infty;\boldsymbol{\rho},\mathbf{v}) = 1 + \frac{i}{\hbar}\int_{-\infty}^{\infty} dt\, \mathbf{d}\cdot\mathbf{E}(t;\boldsymbol{\rho},\mathbf{v}) + \left(\frac{i}{\hbar}\right)^2 \int_{-\infty}^{\infty} dt\, \mathbf{d}\cdot\mathbf{E}(t;\boldsymbol{\rho},\mathbf{v}) \int_{-\infty}^{t} ds\, \mathbf{d}\cdot\mathbf{E}(s;\boldsymbol{\rho},\mathbf{v}). \quad (4.1)$$

$\mathbf{d}$ is the atomic dipole moment, restricted to the space of states of the upper level, and $\mathbf{E}(t;\boldsymbol{\rho},\mathbf{v})$ is the electric field at the origin, produced by a perturber with position $\boldsymbol{\rho}+\mathbf{v}t$. For singly charged ions, this is

$$\mathbf{E}(t;\boldsymbol{\rho},\mathbf{v}) = -e(\boldsymbol{\rho}+\mathbf{v}t)\,|\boldsymbol{\rho}+\mathbf{v}t|^{-3}. \quad (4.2)$$

On the average, the first order term in (4.1) vanishes since we have

$$\int_{-\infty}^{\infty} dt\, \mathbf{E}(t;\boldsymbol{\rho},\mathbf{v}) = -2e\,\varrho^{-2}v^{-1}\boldsymbol{\rho} \quad (4.3)$$

which is odd with respect to the sign of $\boldsymbol{\rho}$. Writing out the second order contribution, we get a sum of four terms containing $(\mathbf{d}\cdot\boldsymbol{\rho})^2$, $(\mathbf{d}\cdot\boldsymbol{\rho})(\mathbf{d}\cdot\mathbf{v})$, $(\mathbf{d}\cdot\mathbf{v})(\mathbf{d}\cdot\boldsymbol{\rho})$, and $(\mathbf{d}\cdot\mathbf{v})^2$, respectively. The first of these is

$$\left(\frac{ie}{\hbar}\right)^2 (\mathbf{d}\cdot\boldsymbol{\rho})^2 \int_{-\infty}^{\infty} dt\,(\varrho^2+v^2t^2)^{-3/2} \int_{-\infty}^{t} ds\,(\varrho^2+v^2s^2)^{-3/2} = \frac{1}{2}\left(\frac{ie}{\hbar}\right)^2 (\mathbf{d}\cdot\boldsymbol{\rho})^2 \left[\int_{-\infty}^{\infty} dt\,(\varrho^2+v^2t^2)^{-3/2}\right]^2 = \left(\frac{ie}{\hbar}\right)^2 \frac{2}{\varrho^4 v^2}(\mathbf{d}\cdot\boldsymbol{\rho})^2. \quad (4.4)$$

The second and third one are odd with respect to $\boldsymbol{\rho}$ again and vanish on the average. The last one vanishes, too:

$$\left(\frac{ie}{\hbar}\right)^2 (\mathbf{d}\cdot\mathbf{v})^2 \int_{-\infty}^{\infty} dt\, t\,(\varrho^2+v^2t^2)^{-3/2} \int_{-\infty}^{t} ds\, s\,(\varrho^2+v^2s^2)^{-3/2} = \frac{1}{2}\left(\frac{ie}{\hbar}\right)^2 (\mathbf{d}\cdot\mathbf{v})^2 \left[\int_{-\infty}^{\infty} dt\, t\,(\varrho^2+v^2t^2)^{-3/2}\right]^2 = 0.$$

Inserting this result into (3.7), we find

$$\Phi(\mathbf{V}) = -\frac{2Ne^2}{\hbar^2}\int d^3v\, v^{-1} f_i(\mathbf{v}+\mathbf{V}) \cdot \int d^3\varrho\, \varrho^{-5}(\mathbf{d}\cdot\boldsymbol{\rho})^2\,\delta(\hat{\varrho}\cdot\boldsymbol{v}). \quad (4.5)$$

If we try to carry the integrations through in (4.5), we meet with the well known divergences of the impact theory at both small and large values of ϱ: For small impact parameters the perturbation expansion breaks down (see (4.4)), while for $\varrho\to\infty$ the S-matrix treatment cannot be used for unshielded perturbers [29]. To remedy these defects, we introduce cutoffs $\varrho_{\min}$ and $\varrho_{\max}$ for the integration over impact parameters. To account for shielding of electrical charges in a plasma, the upper cutoff should be of the order of the Debye length [2, 28, 29], so we set

$$\varrho_{\max} = \varrho_D = \left(\frac{kT}{4\pi N e^2}\right)^{1/2} \quad (4.6)$$

The lower cutoff is usually chosen in such a way that the matrix elements of (4.4) do not exceed unity; a common estimate [2] for the Lyman lines is $n^2\hbar/(m_e v)$, n denoting the principal quantum number of the upper level. In order to keep the calculations as simple as possible we shall not use this value, but its average over perturber velocities according to the Maxwell distribution

$$f_i(\mathbf{u}) = (\pi u_0^2)^{-3/2} e^{-u^2/u_0^2}, \quad u_0^2 = 2kT/m_i . \tag{4.7}$$

This turns out to be

$$\varrho_{\min} = \frac{n^2\hbar}{m_e V} \operatorname{erf}(V/u_0) . \tag{4.8}$$

As the cutoffs would bring in merely a logarithmic dependence on v, we do not expect any major error from this replacement. Moreover, in the case of very heavy, immovable ions u_0 tends to zero and (4.8) becomes the usual lower cutoff.

Collisions with $\varrho > \varrho_{\min}$ are "weak" since they may be treated by perturbation theory. "Strong" collisions with $\varrho < \varrho_{\min}$, on the other hand, have to be processed in a non-perturbative way. A simple estimate of their effects on the line profile follows from the Lorentz-Weisskopf assumption [33] that — on the average — $U(\infty, -\infty; \boldsymbol{\varrho}, \mathbf{v})$ does not contribute to (3.7) for strong collisions. The corresponding portion of the impact operator is

$$\Phi^{(s)}(\mathbf{V}) = -\sqrt{\pi} N \varrho_{\min}^2 \cdot \left[u_0 e^{-V^2/u_0^2} + \frac{\sqrt{\pi}}{2} \frac{2V^2 + u_0^2}{V} \operatorname{erf}(V/u_0) \right], \tag{4.9}$$

it is independent of the direction of $\mathbf{V}$. For an emitter at rest, it becomes

$$\Phi^{(s)}(0) = -\pi N \varrho_{\min}^2 \bar{u} = -2\sqrt{\pi} N \varrho_{\min}^2 u_0 .$$

This is the usual form for strong electron collisions [34], too, if $\varrho_{\min}$ does not depend on v (or u, in that case). For heavy ions or fast emitters all collisions take place with relative velocity V, and we find $\Phi^{(s)} = -\pi N V \varrho_{\min}^2$.

The contribution of weak collisions to the impact operator, $\Phi^{(w)}$, results from (4.5) by inserting (4.7) and limiting the integration over impact parameters to $\varrho_{\min} < \varrho < \varrho_{\max}$. With both cutoffs independent of v, the result may be obtained in closed form (consult the appendix for details). If the z-axis of the coordinate system points into the direction of $\mathbf{V}$, $\mathbf{V} = V\mathbf{e}_z$, one finds

$$\Phi^{(w)}(V\mathbf{e}_z) = -\frac{2\sqrt{\pi} N e^2}{\hbar^2 u_0} \ln \frac{\varrho_{\max}}{\varrho_{\min}} \cdot [A(V/u_0)(d_x^2 + d_y^2) + B(V/u_0)\, d_z^2] \tag{4.10}$$

where the functions A and B are defined by

$$A(x) = x^{-2} e^{-x^2} + \frac{\sqrt{\pi}}{2} x^{-1}(2 - x^{-2}) \operatorname{erf}(x) ,$$
$$B(x) = -2x^{-2} e^{-x^2} + \sqrt{\pi} x^{-3} \operatorname{erf}(x) . \tag{4.11}$$

Formula (4.10), together with (4.11), clearly displays that the impact operator for a moving emitter depends not only on the magnitude of the emitter velocity but on its direction as well — thus reflecting the anisotropy of the plasma as seen from the emitter. As a result, $I_S^{\parallel}$ and $I_S^{\perp}$ differ from one another for moving radiators, and it is indeed necessary to distinguish between these two profiles. It is only for emitters flying much slower than the perturbers, i.e., $V/u_0 \simeq 0$, that A and B are equal: $A(0) = B(0) = 4/3$. Then the impact operator for an atom at rest [28, 29] (perturbed by weak collisions only) follows from (4.10):

$$\Phi^{(w)}(\mathbf{V} = 0) = -\frac{8\sqrt{\pi} N e^2}{3\hbar^2 u_0} \ln \frac{\varrho_{\max}}{\varrho_{\min}} \mathbf{d} \cdot \mathbf{d} . \tag{4.12}$$

This expression applies to electron broadening, too, if we replace u_0 by $(2kT/m_e)^{1/2}$, the characteristic thermal speed of electrons. But this is greater than u_0 by a factor of at least about 40, so electron broadening may be neglected.

Figure 1 shows a graph of the functions A and B. Most interesting is the fact that $B(V/u_0)$ falls off much more rapidly than $A(V/u_0)$ if u_0 tends to zero or V/u_0 goes to infinity. Hence, in that limit, the contribution of the component of the atomic

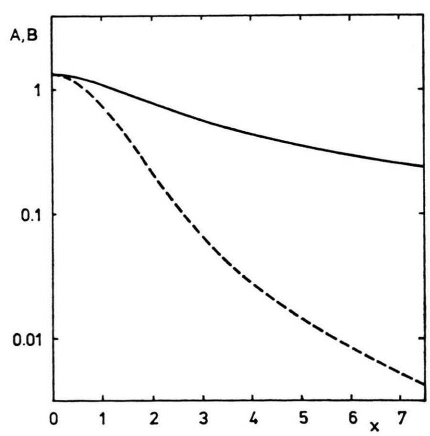

Fig. 1. The functions A (———) and B (- - - -) defined in (4.11). At $x = 0$, both functions have the value 4/3. Asymptotically, for $x \to \infty$, we find $A(x) \sim \sqrt{\pi}/x$ and $B(x) \sim \sqrt{\pi}/x^3$.

dipole moment in the direction of $\mathbf{V}$ vanishes (at least to the second order of perturbation theory), and the dependence of $\Phi^{(w)}$ on the direction of the emitter velocity becomes most conspicious. Therefore we are going to consider this case of immovable "static" ions in more detail now.

5. Impact Broadening by Static Ions

At a first glance, "impacts by static ions" seems to be a contradiction in terms. It has been emphasized, however, that what matters is the relative perturber-emitter velocity, so the idea of a moving emitter colliding with an ion at rest has nothing strange about it. Here we shall investigate this extreme case more closely.

For $u_0 \to 0$, we find from (4.9) and (4.8) for strong collisions

$$\Phi_{\mathrm{si}}^{(\mathrm{s})}(\mathbf{V}) = -\pi N V \varrho_{\min}^2 = -\pi N \frac{n^4 \hbar^2}{m_e^2 V}, \qquad (5.1)$$

the subscript "si" standing for "static ions". The weak collisions, on the other hand, lead to

$$\Phi_{\mathrm{si}}^{(\mathrm{w})}(V \mathbf{e}_z) = -\frac{2\pi N e^2}{\hbar^2 V}(d_x{}^2 + d_y{}^2) \ln \frac{\varrho_{\max}}{\varrho_{\min}} \qquad (5.2)$$

which is the appropriate limit of (4.10) and (4.11). These results might have been obtained more easily directly from (3.7) and (4.5) by inserting $\delta(\mathbf{u})$ for $f_i(\mathbf{u})$.

A closer inspection of (5.2) reveals a problem for small emitter velocities: As V goes to zero, $\varrho_{\min} = n^2 \hbar/(m_e V)$ increases without bound and eventually becomes greater than $\varrho_{\max}$ which does not vary with V. This is no surprise, since the impact theory must break down for slowly moving emitters in an environment of stationary ions. The impact theory of electron broadening has to face a similar difficulty for slow electrons [34]. There, it is argued that those electrons for which $\varrho_{\min}$ exceeds $\varrho_{\max}$ constitute only a small fraction if the impact theory is valid at all and hence may be neglected in the determination of the line core. The argument here is just the same: Due to the rough estimate of strong collision effects with the help of the Lorentz-Weisskopf approximation, we have to demand that $\Phi_{\mathrm{si}}^{(\mathrm{w})}$ is the dominating term in $\Phi_{\mathrm{si}} = \Phi_{\mathrm{si}}^{(\mathrm{w})} + \Phi_{\mathrm{si}}^{(\mathrm{s})}$. Estimating a typical matrix element of $\mathbf{d}^2$ by $n^4 e^2 a_0{}^2$ ($a_0 = \hbar^2/(m_e e^2)$ is Bohr's radius) [35], we find that $\Phi_{\mathrm{si}}^{(\mathrm{w})}$ is of the order of

$$2\pi N n^4 \hbar^2 m_e^{-2} V^{-1} \ln(\varrho_{\max}/\varrho_{\min}) .$$

On comparison to (5.1), this validity condition for the impact theory turns out to be $\ln(\varrho_{\max}/\varrho_{\min}) \gg 1$. for typical values of V, for example V_0. But the other way round, this says that $\varrho_{\min} > \varrho_{\max}$ occurs only for $V \ll V_0$ which is very unlikely with a Maxwell distribution. (The probability to find an atom with speed $V < V_0/10$ is less than 0.1%.) Hence, we simply disregard these radiators.

On the other hand, $\ln(\varrho_{\max}/\varrho_{\min}) \gg 1$ sets an upper limit on the plasma densities (at fixed temperature) for which the perturbation form of the impact theory may be expected to give reliable results in the line core:

$$\frac{(m_e k T)^2}{2\pi M (n^2 e \hbar)^2} \gg N , \qquad (5.3)$$

again inserting $\varrho_{\min}(V_0)$. Because of the transition from the logarithm to its argument, "$\gg$" really means "much greater than" in (5.3). (Note that x has to be about 10^4 for $\ln(x) = 10$.) For L_α ($n=2$) and a plasma temperature of 10^4 K, (5.3) says $N \ll 4 \cdot 10^{16}$ cm^{-3}, so we do not expect the impact theory for ions to be valid at densities above 10^{13} cm^{-3}.

The full impact operator is proportional to V^{-1} if we neglect the weak logarithmic dependence on V (through $\varrho_{\min}$) of $\Phi_{\mathrm{si}}^{(\mathrm{w})}$ for the moment. Therefore, the Stark profiles $I_S{}^{\parallel}$ and $I_S{}^{\perp}$, which are

$$\begin{aligned} I_S{}^{\parallel}(\omega|V) &\propto -\operatorname{Re} \sum_{a,a'} \langle n a | d_x | 1\rangle \langle 1 | d_x | n a'\rangle \\ &\quad \cdot \langle n a' | [i \Delta\omega + \Phi(V \mathbf{e}_z)]^{-1} | n a\rangle , \\ I_S{}^{\perp}(\omega|V) &\propto -\operatorname{Re} \sum_{a,a'} [\langle n a | d_x | 1\rangle \langle 1 | d_x | n a'\rangle \\ &\quad + \langle n a | d_z | 1\rangle \langle 1 | d_z | n a'\rangle] \qquad (5.4) \\ &\quad \cdot \langle n a' | [i \Delta\omega + \Phi(V \mathbf{e}_z)]^{-1} | n a\rangle \end{aligned}$$

for the Lyman lines, depend strongly on the emitter speed. ($|n a\rangle$ are the states of the upper level — each of these is assumed to be found with the same probability $1/n^2$ — and $|1\rangle$ is the hydrogen ground state.) It is evident that the "one and only" Stark profile (independent of $\mathbf{V}$) which has been evaluated by most calculations up to now is merely a fictious quantity. Beside this, (5.4) shows that there is a distinct statistical correlation of Stark and Doppler broadening at least in the impact limit for ions: Stark widths are greater for slowly moving atoms than they are for fast ones, while the reverse is true for the Doppler width as function of V.

Theoretically, this constitutes a rather simple and transparent example of joint Stark-Doppler broadening. Yet, in practice the velocity dependence of Stark broadening and its correlation with Doppler broadening are completely insignificant at the low densities where we may use the impact theory for ions. This is due to the overwhelming importance of Doppler broadening in the line core, whether relative emitter-ion motion is accounted for [17] or not [36]. (The far line wing always has the Stark profile which falls off more slowly than the Doppler profile, but then the impact theory ceases to be valid.) Nevertheless, our result has some bearing for higher densities as there must be a smooth transition from purely dynamical (impact) broadening at very low densities to static (Holtsmark) broadening at very high densities. Indeed, distinct effects of emitter-ion motion have been found in the line centres, experimentally [4—11] and theoretically [17, 18], for intermediate plasma densities of about 10^{16} cm^{-3} where Doppler broadening is less important than Stark broadening. It is in these circumstances that the dependence of Stark broadening on the radiator velocity cannot be neglected in theoretical calculations without introducing another source of possible errors. (Results obtained from an approximate treatment utilizing the "model microfield method" indicate that there is a still appreciable variation of Stark broadening with the emitter speed at these densities [37].)

With the impact theory we have elaborated here it is of course impossible to investigate these questions. Nevertheless, some impact broadening by ions is found even at high plasma densities, namely for very fast emitters from the "tail" of the Maxwell distribution. Their number is too small in a dense plasma to exercise any measurable influence on the final line profile, but the impact profiles given here constitute the correct limit of the speed dependent Stark profiles $I_S^{\parallel}$ and $I_S^{\perp}$ as V tends to infinity [37]. (Consequently, Stark widths do not increase forever with radiator speed though this appears to be true for $V \simeq V_0$ in dense plasmas [15, 37].) Apart from this, the main merit of the impact theory is the clear demonstration of the velocity dependence of Stark broadening. To complete its representation, we shall at last consider L_α in more detail, which is the most simple line for the theory.

The upper level of L_α is fourfold degenerate, hence all operators like Φ_{si} are represented by 4×4-matrices. As basis for the representation we use the angular momentum eigenstates $|2lm\rangle$, ordered in this way: $|200\rangle$, $|210\rangle$, $|211\rangle$, $|21-1\rangle$. Then we find a diagonal matrix for $d_x^2 + d_y^2$:

$$d_x^2 + d_y^2 = 9e^2 a_0^2 \,\mathrm{diag}\,(2, 0, 1, 1)\,. \tag{5.5}$$

Accordingly, Φ_{si} is diagonal in this representation, too, and the inversion of $i\Delta\omega + \Phi_{si}$ in (5.4) is easily done. Electric dipole transitions to the ground state $|1\rangle = |100\rangle$ are allowed from $|210\rangle$, $|211\rangle$, and $|21-1\rangle$ because of

$$|z_{210}^{100}|^2 = 2\,|x_{211}^{100}|^2 = 2\,|x_{21\,\text{-}1}^{10\;\,0}|^2 \neq 0\,.$$

Hence the Stark profiles are

$$I_S^{\parallel}(\omega\,|\,V) = -\frac{1}{2\pi} \cdot \mathrm{Re} \sum_{m=1,-1} \langle 21m|\, i\Delta\omega + \Phi_{si}(V\mathbf{e}_z)\,|21m\rangle^{-1}\,, \tag{5.6}$$

$$I_S^{\perp}(\omega\,|\,V) = -\frac{1}{4\pi} \cdot \mathrm{Re} \sum_{m=-1}^{1} (2-|m|)\,\langle 21m|\, i\Delta\omega + \Phi_{si}(V\mathbf{e}_z)\,|21m\rangle^{-1}\,.$$

From (5.1), (5.2), and (5.5), the impact operator is found to be

$$\Phi_{si}(V\mathbf{e}_z) = -\,\mathrm{diag}\,(2a+b, b, a+b, a+b)\,, \tag{5.7}$$

a and b depending on V. Inserting this into (5.6), the Stark profiles are

$$I_S^{\parallel}(\omega\,|\,V) = \frac{1}{\pi}\,\frac{a+b}{(\Delta\omega)^2 + (a+b)^2}\,, \tag{5.8}$$

$$I_S^{\perp}(\omega\,|\,V) = \frac{1}{2\pi} \cdot \left[\frac{a+b}{(\Delta\omega)^2+(a+b)^2} + \frac{b}{(\Delta\omega)^2+b^2}\right].$$

Measuring the plasma density in units of cm^{-3}, $N = \boldsymbol{N^*}$ cm^{-3}, the temperature in K, $T = \boldsymbol{T^*}$ K, and the speed in multiples of V_0, $V = \boldsymbol{V^*} V_0$, we find for hydrogen

$$a = 3.81 \cdot 10^{-19}\, \boldsymbol{N^*} \boldsymbol{T}^{*-1/2} \boldsymbol{V}^{*-1}\, \omega_{21} \cdot \ln(1.92 \cdot 10^4\, \boldsymbol{T^*} \boldsymbol{V^*} \boldsymbol{N}^{*-1/2})\,, \qquad b = 3.39 \cdot 10^{-19}\, \boldsymbol{N^*} \boldsymbol{T}^{*-1/2} \boldsymbol{V}^{*-1}\, \omega_{21} \tag{5.9}$$

with $\omega_{21} = 3e^2/(8\hbar a_0)$, the central frequency of L_α. As validity condition for the impact theory we had to require that a is much greater than b, so the

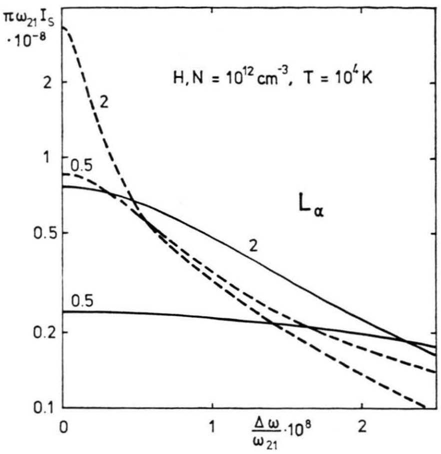

Fig. 2. Speed dependent Stark profiles $I_S^{\parallel}$ (———) and $I_S^{\perp}$ (– – – –) of L_α emitted by hydrogen from a plasma with infinitely heavy ions ($N = 10^{12}\,\mathrm{cm}^{-3}$, $T = 10^4$ K). Parameter is the emitter speed V, given in multiples of the characteristic thermal velocity $V_0 = \sqrt{2/k\,T M_H}$.

two Lorentz profiles appearing in (5.8) differ greatly in width and $I_S^{\perp}$ exhibits a pronounced "shoulder" (Figure 2). (This is not the shoulder of "static" profiles which are composed of an unshifted central component together with two symmetrically shifted side components.)

In Fig. 2, both Stark profiles of L_α are shown for radiator speeds $V_0/2$ and $2\,V_0$; it provides a pictorial illustration of the velocity dependence of Stark broadening and of the differences between $I_S^{\parallel}$ and $I_S^{\perp}$. It should be annotated that neither $V_0/2$ nor $2\,V_0$ are extremely unlikely thermal speeds: Still about 5% of all atoms have a speed less than $V_0/2$, and another 5% move faster than $2\,V_0$. The "completed collision condition" [2] mentioned at the end of section 3 is quite well fulfilled for the speeds and frequencies shown as we have $V_0/\varrho_{\max} \simeq 10^{-7}\,\omega_{21}$ for the plasma conditions of Figure 2. We have stressed repeatedly that Stark broadening by ion impacts is merely of theoretical interest as it is completely hidden by Doppler broadening. Indeed, the Doppler widths $\Delta\omega_D(V) = \omega_{21}\,V/c$ are far out of scale in Figure 2; we find $10^8 \Delta\omega_D(V_0)/\omega_{21} \simeq 4300$, for instance. Accordingly, Doppler broadening cannot be neglected in the full line profile. Despite of this we shall calculate the correct "pure" Stark profile (by inserting (5.8) into (2.6), but disregarding the Doppler shift there) and compare it to the commonly used approximate form obtained for an atom at rest surrounded by fictious ions with the reduced atom-ion mass (which is the atomic mass for static ions). Though none of these profiles has physical significance, their comparison will help to accentuate the influence of emitter motion on Stark broadening once more.

The correct „pure" Stark profile corresponding to (5.8) is

$$\tilde{I}_S(\omega) = \int_{V_{\min}}^{\infty} \mathrm{d}V\, 4\pi\, V^2 f_M(V) \cdot \frac{1}{3\pi}\left[2\frac{a+b}{(\Delta\omega)^2 + (a+b)^2} + \frac{b}{(\Delta\omega)^2 + b^2}\right], \tag{5.10}$$

where a and b vary with V according to (5.9), and $V_{\min}$ is defined by $\varrho_{\min}(V_{\min}) = \varrho_{\max}$. The "usual" impact operator Φ'_{si}, on the other hand, is the average over $\mathbf{V}$ of $\Phi_{si}(-\mathbf{V})$ with the distribution function $f_M(V)$ (which is the distribution of *perturber velocities* here). Due to the angular average included, the result is invariant under rotations. Using (5.7) and again neglecting velocities below $V_{\min}$, we find

$$\Phi'_{si} = -\operatorname{diag}(3a' + b', a' + b', a' + b', a' + b') \tag{5.11}$$

with the following contributions of weak and strong collisions ($\boldsymbol{M}^* = M/M_H$):

$$a' = 1.43 \cdot 10^{-19}\, \boldsymbol{N}^* \boldsymbol{M}^{*1/2}\, \boldsymbol{T}^{*-1/2}\, \omega_{21} \cdot E_1(V_{\min}^2/V_0^2)\,,$$
$$b' = 3.82 \cdot 10^{-19}\, \boldsymbol{N}^* \boldsymbol{M}^{*1/2}\, \boldsymbol{T}^{*-1/2}\, \omega_{21} \cdot \exp(-V_{\min}^2/V_0^2)\,. \tag{5.12}$$

Except for the appearance of the atomic mass in (5.12), Φ'_{si} is identical with the impact operator for electron broadening [34, 38]. The discussion at the beginning of this section has shown that we must have $V_{\min} \ll V_0$, so the appropriate expansions may be used for the exponential integral and the exponential function in (5.12), i.e., $E_1(V_{\min}^2/V_0^2) \simeq -\gamma - \ln(V_{\min}^2/V_0^2) \simeq 2\ln(\tfrac{3}{4}\,V_0/V_{\min})$ and

$$\exp(-V_{\min}^2/V_0^2) \simeq 1$$

($\gamma \simeq 0.5772$ is Euler's constant). Due to the rotational invariance of Φ'_{si}, the "usual" Stark profile is a simple Lorentz profile:

$$I_S'(\omega) = \frac{1}{\pi}\,\frac{a' + b'}{(\Delta\omega)^2 + (a' + b')^2}\,. \tag{5.13}$$

In Fig. 3, $\tilde{I}_S$ and I_S' are shown for a plasma with $\boldsymbol{N} = 10^{12}$, $\boldsymbol{T} = 10^4$. The two profiles are clearly different: While $\tilde{I}_S$ is the sum of a broad and a

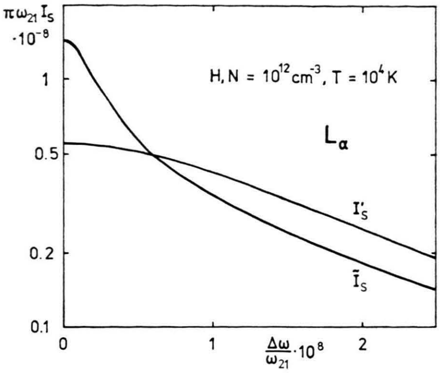

Fig. 3. "Pure" Stark profiles $\tilde{I}_S$ and $\tilde{I}_S'$ of L_α for the same plasma conditions as in Figure 2. To emphasize the importance of a correct treatment of the dependence of Stark broadening on the emitter velocity, Doppler broadening is disregarded in both profiles. $\tilde{I}_S$ is the "correct" Stark profile obtained as velocity average of the *line profile*, while the "usual" Stark profile I_S' results from the velocity average of the *impact operator*.

narrow part, the latter one is missing in I_S'. The reason for its appearance in $\tilde{I}_S$ is the fact that — in the case considered here — only strong collisions affect the component of $\langle \mathbf{d}(t|\mathbf{V})\rangle_p$ in the direction of $\mathbf{V}$, while the components perpendicular to $\mathbf{V}$ "feel" weak collisions as well. With increasing plasma density this anisotropy will certainly become less drastic. (It vanishes completely for „static" microfield broadening, and electron broadening blurs it, too.) Therefore, we may assume that the approximation of considering an emitter at rest and perturbers with the reduced mass will work better at higher densities than it does work here. It is, however, hard to estimate the errors introduced by this.

6. Summary and Conclusions

The experimental discovery of "ion dynamical effects" in the cores of plasma broadened hydrogen lines [3—11] has demonstrated the untrustworthiness of the "static ions approximation" [2] for that case. Various theoretical approaches [12—19] have since been elaborated to take account of relative emitter-ion motion, but none of them — with the exception of Griem's latest treatment [15] of L_α and "model microfield calculations" of the author [37] which are not yet published — tries to make allowance for the influence of emitter motion on hydrogen Stark broadening. This is the more astonishing as already the comparatively simple "classical path theory" of Stark broadening for moving radiators [20, 21] compels to consider Stark profiles which depend on the emitter velocity.

As seen from a moving radiator the plasma surrounding it is no longer isotropic. To take proper care of this, two kinds of speed dependent Stark profiles have been defined in Sect. 2 which describe the emission of light parallel and perpendicular to the radiator velocity in a frame of reference moving with the same velocity. From these the correct full line profile including Doppler broadening as well is obtained according to formula (2.6).

To give a transparent example of velocity dependent Stark broadening, the impact theory [2, 28, 29] of hydrogen line broadening by plasma ions has been evaluated in Sects. 3 to 5. Ion impact broadening of the line core is found only at extremely low plasma densities where Doppler broadening is much more important, so it is of no practical interest. Nevertheless, a theoretical investigation seems to be worth while: Impact broadening is purely "dynamical" (as opposed to broadening by "static" plasma microfields [1, 2]) and most pronounced effects of emitter motion may be expected in this extreme case, especially with heavy ions which are almost immovable themselves.

In the impact theory, the anisotropy brought about by the motion of the radiating atom makes itself felt in the probability density for atom-ion collisions (Sect. 3). For that reason, the ion impact operator depends on the atomic velocity $\mathbf{V}$. This becomes most obvious if second order perturbation theory is used for its evaluation as has been done in Section 4. Owing to this, the impact Stark profiles depend on radiator speed V, and for very heavy "static" ions their half widths are essentially proportional to $1/V$ as has been shown in Section 5. Moreover, the profiles corresponding to emission parallel or perpendicular to $\mathbf{V}$ are found to be all different. Hence, the conception of velocity dependent Stark profiles is not void and has to be utilized in a correct treatment of "ion dynamical effects". (For that reason, by the way, "pure" Stark profiles extracted from experimental data by deconvolution of a Gaussian Doppler profile lack any physical meaning in general.)

As the impact theory of ion Stark broadening applies to almost all atoms at low plasma densities it may be used to calculate the line cores in that case. Some impact broadening by ions, however, is found even in dense plasmas, namely for radiators flying very fast (the number of which is too small then to exert any influence on the line profile). Therefore, at any plasma density the impact profiles given here constitute the correct limit of the speed dependent Stark profiles $I_S^{\parallel}(\omega|V)$ and $I_S^{\perp}(\omega|V)$ as V goes to infinity. In fact, model microfield profiles which approximately account for emitter motion tend toward impact profiles in this limit as will be shown elsewhere [37].

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Appendix

To infer (4.10) from (4.5), we use polar coordinates for the components of $\boldsymbol{\rho}$ and $\mathbf{v}$:

$$\boldsymbol{\rho} = \varrho(\boldsymbol{e}_x \sin\vartheta\cos\varphi + \boldsymbol{e}_y \sin\vartheta\sin\varphi + \boldsymbol{e}_z\cos\vartheta)\,,$$
$$\mathbf{v} = v(\boldsymbol{e}_x \sin\alpha\cos\beta + \boldsymbol{e}_y \sin\alpha\sin\beta + \boldsymbol{e}_z\cos\alpha)\,; \tag{A.1}$$

and α are the angles formed by $\mathbf{V} = V\boldsymbol{e}_z$ and $\boldsymbol{\rho}$ or $\mathbf{v}$, respectively. With the Maxwell distribution (4.7) of perturber velocities, the impact operator for weak collisions becomes

$$\Phi^{(\mathrm{w})}(V\boldsymbol{e}_z) = -\frac{2Ne^2}{\hbar^2}\int_0^\infty \mathrm{d}v\, v\int_0^\pi \mathrm{d}\alpha\sin\alpha\int_0^{2\pi}\mathrm{d}\beta\int_{\varrho_{\min}}^{\varrho_{\max}}\mathrm{d}\varrho\,\varrho^{-1}\int_0^\pi \mathrm{d}\vartheta\sin\vartheta \cdot\int_0^{2\pi}\mathrm{d}\varphi(\mathbf{d}\cdot\hat{\varrho})^2(\pi u_0^2)^{-3/2}\exp\left(-\frac{v^2+V^2+2vV\cos\alpha}{u_0^2}\right) \cdot\delta[\sin\alpha\sin\vartheta\cos(\beta-\varphi)+\cos\alpha\cos\vartheta]\,. \tag{A.2}$$

The β-integration relates to nothing else but the δ-function, and as it extends over the full period of the cosine it is possible to replace $\cos(\beta-\varphi)$ by $\cos\beta$. But then, the φ-integration refers to $(\mathbf{d}\cdot\hat{\varrho})^2$ only, and none of the mixed terms survives. Moreover, the ϱ-integration may be done immediately. Using $t = v/u_0$ as integration variable instead of v and defining $\xi = V/u_0$, we thus get the form (4.10) for $\Phi^{(\mathrm{w})}$:

$$\Phi^{(\mathrm{w})}(V\boldsymbol{e}_z) = -\frac{2\sqrt{\pi}\,Ne^2}{\hbar^2 u_0}\ln\frac{\varrho_{\max}}{\varrho_{\min}}[A(\xi)(d_x^2+d_y^2)+B(\xi)d_z^2]\,. \tag{A.3}$$

It remains to calculate A and B:

$$\left.\begin{matrix}A(\xi)\\B(\xi)\end{matrix}\right\} = \frac{1}{\pi}\int_0^\infty \mathrm{d}t\,t\int_0^\pi\mathrm{d}\alpha\sin\alpha\int_0^\pi\mathrm{d}\vartheta\sin\vartheta\left\{\begin{matrix}\sin^2\vartheta\\2\cos^2\vartheta\end{matrix}\right\} \cdot\exp\{-(t^2+\xi^2+2t\xi\cos\alpha)\}\int_0^{2\pi}\mathrm{d}\beta\,\delta(\sin\alpha\sin\vartheta\cos\beta+\cos\alpha\cos\vartheta)\,. \tag{A.4}$$

The β-integral is conveniently split into the integrals from 0 to π and from π to 2π which both give the same result; it is found to be

$$\int_0^{2\pi}\mathrm{d}\beta\,\delta(\sin\alpha\sin\vartheta\cos\beta+\cos\alpha\cos\vartheta) = 2\,\frac{\Theta(\sin^2\alpha-\cos^2\vartheta)}{(\sin^2\alpha-\cos^2\vartheta)^{1/2}}\,. \tag{A.5}$$

Now the integration is easily done, leading to

$$\left.\begin{matrix}A(\xi)\\B(\xi)\end{matrix}\right\} = \int_0^\infty\mathrm{d}t\,t\int_0^\pi\mathrm{d}\alpha\sin\alpha\exp\{-(t^2+\xi^2+2t\xi\cos\alpha)\}\cdot\left\{\begin{matrix}(1+\cos^2\alpha)\\2\sin^2\alpha\end{matrix}\right.\,. \tag{A.6}$$

As the next step, it is commendable to do the t-integral first — this is a facilitation brought about by choosing ϱ_{max} and ϱ_{min} independent of v. We find

$$\int_0^\infty dt\, t \exp\{-t^2 - 2t\,\xi \cos\alpha\} = \frac{1}{2} - \frac{\sqrt{\pi}}{2}\,\xi \cos\alpha \exp\{\xi^2 \cos^2\alpha\}\,\mathrm{erfc}(\xi\cos\alpha)\,. \tag{A.7}$$

Inserting this into (A.6), and switching to $\cos\alpha$ as integration variable, we can do the α-integral, too, and obtain (4.11).

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